10-1 Complex Numbers

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Pure Imaginary Numbers

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

And so on...

Any number of the form ***bi*** is a pure imaginary number.

$$\left\{\begin{matrix} i^1 = i \\ i^2 = -1 \\ i^3 = -i \\ i^4 = 1 \end{matrix}\right\}$$ Cyclical

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Complex Numbers

Any number of the form ***a + bi*** is a complex number where $a, b \in \mathbb{R}$. A complex number has both a real and imaginary component.

Notation	3+2i	3+2i
a + bi	3	3+0i
(a,b)	2i	0+2i

Everything is a complex Number!

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Algebraic Structures of Complex Numbers

Addition and Subtraction

Ex1. Simplify

1. (3+2i) + (-8+3i) $-5+5i=5(i-1)$

2. (-4+7i)-(3-2i) $-7+9i$

Algebraic Structures of Complex Numbers

Multiplication

Ex2. Simplify

1. (3+2i)·(4-6i) = 12+8i−18i+12 = 24−10i

2. (3-2i)·(3+2i) = 9+4 = 13+0i = 13

Complex Conjugates

z = complex number

z*= complex conjugate of z

Two complex numbers are conjugates iff
$z \cdot z^* \in \mathbb{R}$

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Algebraic Structures of Complex Numbers

Division

Ex3. Simplify

1. $\frac{3}{i} \cdot \frac{i}{i} = \frac{3i}{-1}$
$= -3i$

2. $\frac{4}{5i}$

3. $\frac{3+2i}{7i}$

4. $\frac{6}{4-3i}$

5. $\frac{3-8i}{5+2i}$

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Properties of Conjugates

- $(z^*)^* = z$
- $z^* = z$ if $z \in \mathbb{R}$
- $(z_1 + z_2)^* = z_1^* + z_2^*$
- $(-z)^* = -(z)^*$
- $(z^{-1})^* = (z^*)^{-1}$ if $z \neq 0$

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The FUNdamental Theorem of Algebra

A polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0$

With real or complex coefficients $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$

has *n* zeros.

Conjugate Root Theorem

A polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0$

With real or complex coefficients $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$

Has complex root z, then its conjugate z* is also a root of f(x)

Ex4. Given that 4+5i is a root of the polynomial function $f(x)=x^3 - 6x^2 + 25x + 82$, find all the remaining roots and check your answers on the GC.

root: $4+5i$, $4-5i$

$(x-(4+5i))(x-(4-5i))$

$(x-4-5i)\cdot(x-4+5i)$

$x^2-4x+5ix-4x+16-20i-5xi+20i+25$

$x^2-8x+41$

$$\begin{array}{r} x+2 \\ x^2-8x+41 \overline{\smash{)}\, x^3-6x^2+25x+82} \\ -(x^3-8x^2+41x+0) \\ \hline 0+2x^2-16x+82 \\ -(2x^2-16x+82) \\ \hline 0+0+0 \end{array}$$

$(x-4-5i)(x-4+5i)(x+2)$

HW pg 438 #7-9, 11, 13, 15, 21-23, 26, 28, 31, 33, 38, 41, 43, 44, 48, 50

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